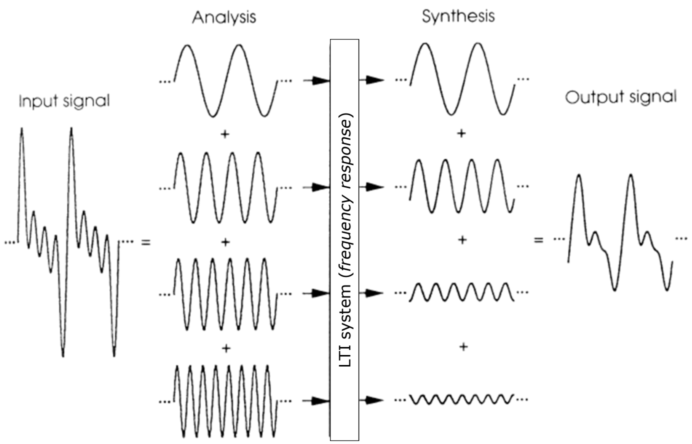
Signals & Systems for Speech & Hearing

Week V

Signals through Systems

Crucial ideas

- *Any* signal can be constructed as a sum of sine waves
- In a *linear time-invariant* (LTI) system, the response to a sinusoid is the same whether it is on its own, or as one component of a complex signal
  – No interaction of components
- An LTI system *never* introduces frequency components not present in the input
  – a sinusoidal input gives a sinusoidal output of the same frequency
- Hence, the output is the *sum* of the individual sinusoidal responses to each individual sinusoidal component of the input



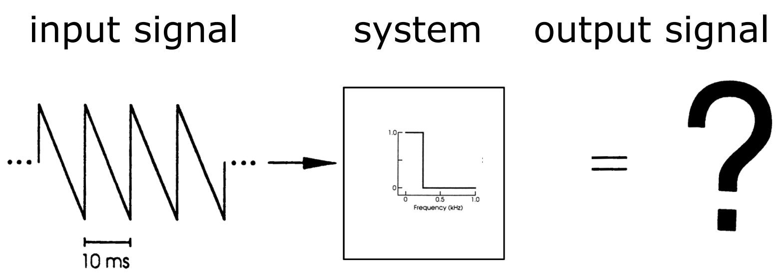
waveform → *spectrum* *spectrum* → *waveform*

Fourier analysis Fourier synthesis

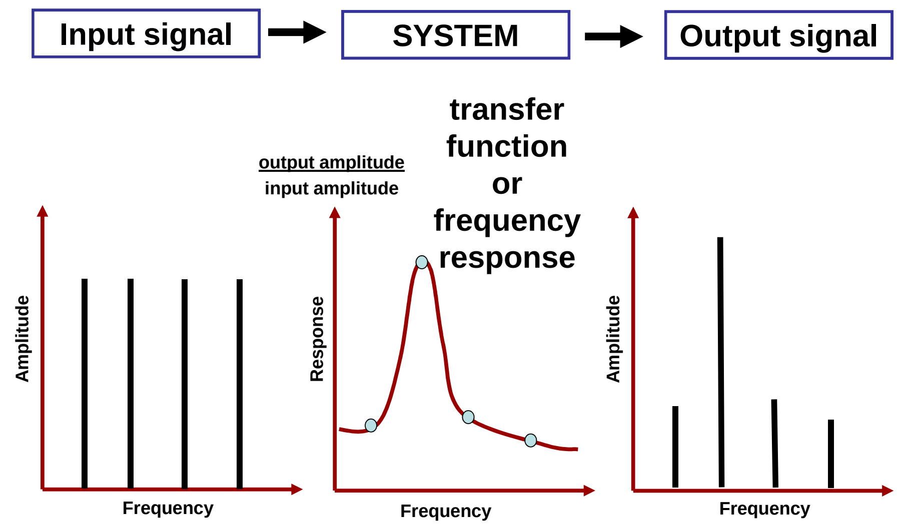
Six steps to determining system output to any particular input

1. Obtain the system's amplitude response
2. Obtain the system's phase response
3. Analyse the waveform to obtain its spectrum (amplitude and phase)
4. Calculate the output amplitude of each component sinusoid in the input spectrum
5. Calculate the output phase of each sinusoid
6. Sum the output component sinusoids

A particular example



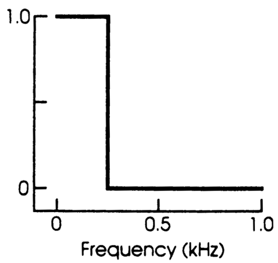
Step 1: Determinining a frequency response



Step 1: Measure the system's response

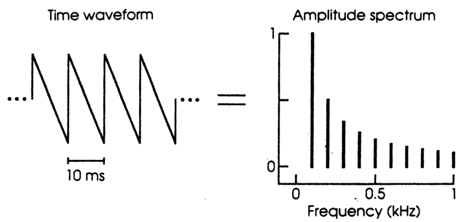
For example, by using sinewaves of different frequencies (as for the acoustic resonator)

Here the response has a gain of
1 for frequencies up to 250 Hz
0 for frequencies above 250 Hz



Assume phase response is a phase shift of zero degrees everywhere

Step 2: Sawtooth amplitude spectrum



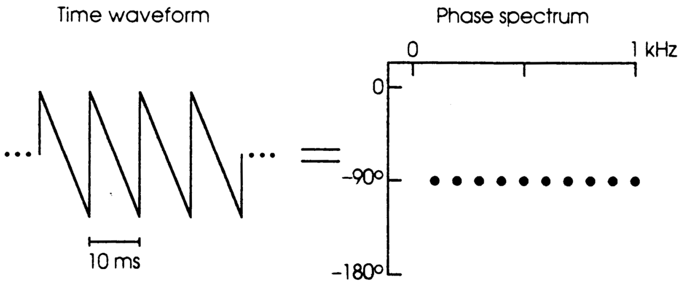
$A(n) = A(1)/n$

(A is the amplitude of a harmonic, index n is harmonic number)

$A(1)$ is for this example 1 volt

Sawtooth phase spectrum

All components have a phase of -90^o (relative to a cosine)



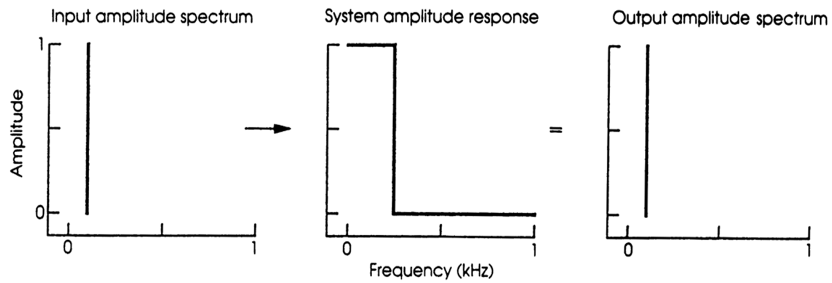
Remember!

- Response = Output amplitude/Input amplitude
- So on linear scales …
  - Output amplitude = Response x Input amplitude
- But on dB (logarithmic) scales
  - Output amplitude = Response + Input amplitude
  - because log(a x b) = log(a) + log(b)
- For phase
  - Output phase = Response phase + Input phase

Response to harmonic 1 (100 Hz)

Input amplitude	Response gain	Output amplitude
1 V	1	?
Input phase	Phase shift of response	Output phase
-90	0	?

Graph of signal - system - output for harmonic 1

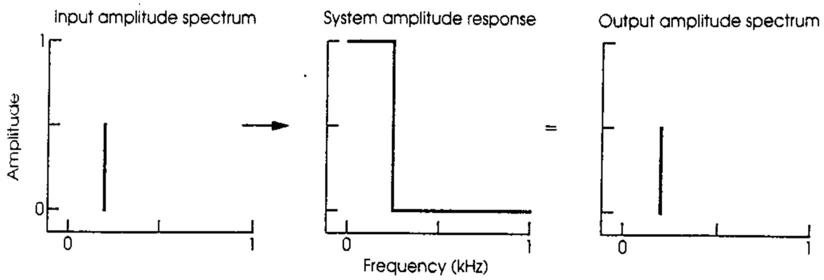


1 V x 1 = 1 V

Response to harmonic 2 (200 Hz)

Input amplitude	Response gain	Output amplitude
1/2 V	1	?
Input phase	**Phase shift of response**	**Output phase**
-90	0	?

Graph of signal - system - output for harmonic 2

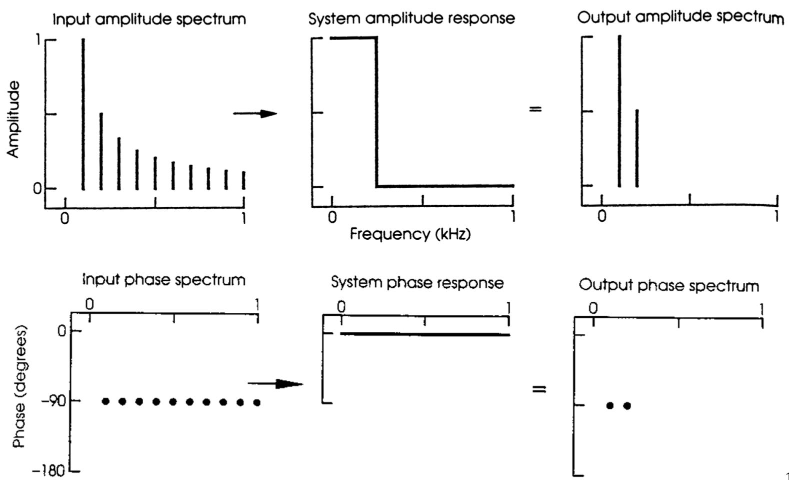


0.5 V x 1 = 0.5 V

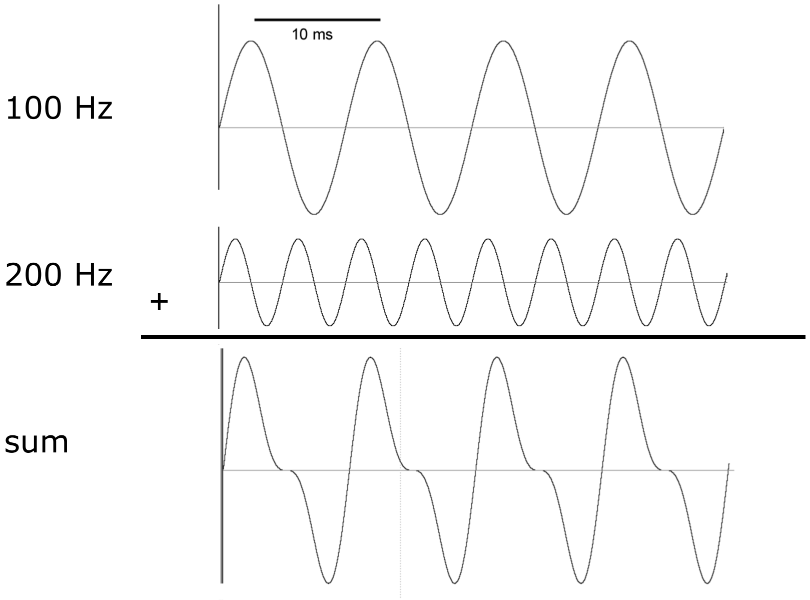
Response to harmonic 3 (300 Hz)

Input amplitude	Response gain	Output amplitude
1/3 V	0	?
Input phase	**Phase shift of response**	**Output phase**
-90	0	?

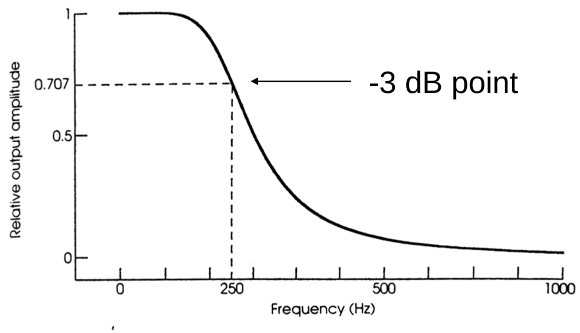
Response to whole signal



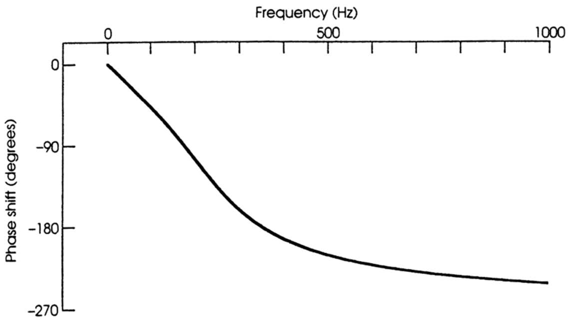
Waveform of output



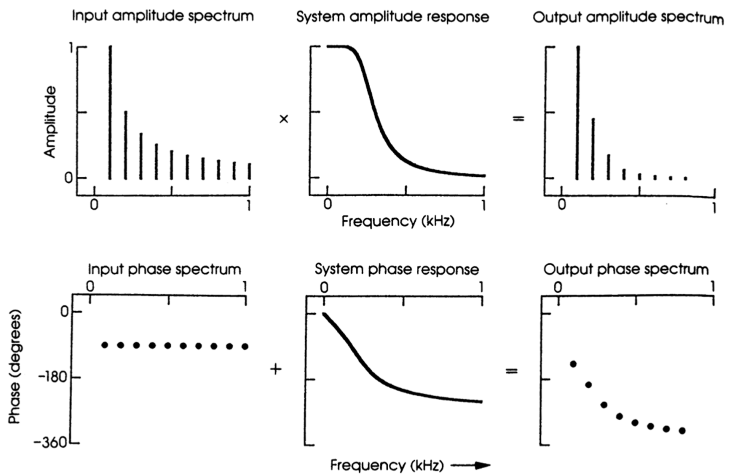
A realistic amplitude response



A phase response



Sawtooth Wave: Signal - System - Output

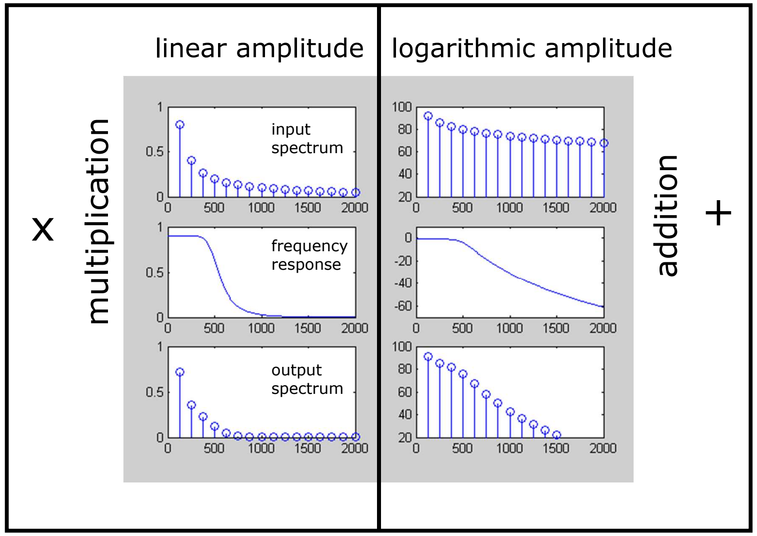


Note that multiplications are done ‘all at once’

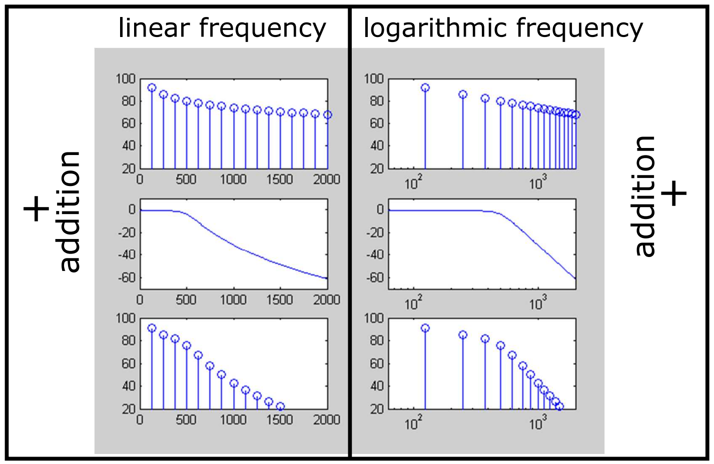
Time waveform response for sawtooth wave

Note how different it looks from the output waveform of the ideal low-pass filter:

Linear vs. logarithmic *amplitude* scales

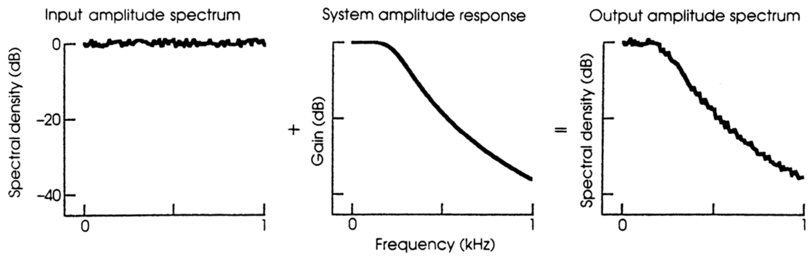


Linear vs. logarithmic *frequency* scales



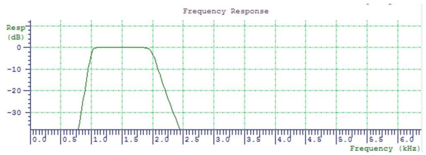
Logarithmic amplitude scales matter for calculations.
Logarithmic frequency scales are a matter of convenience.

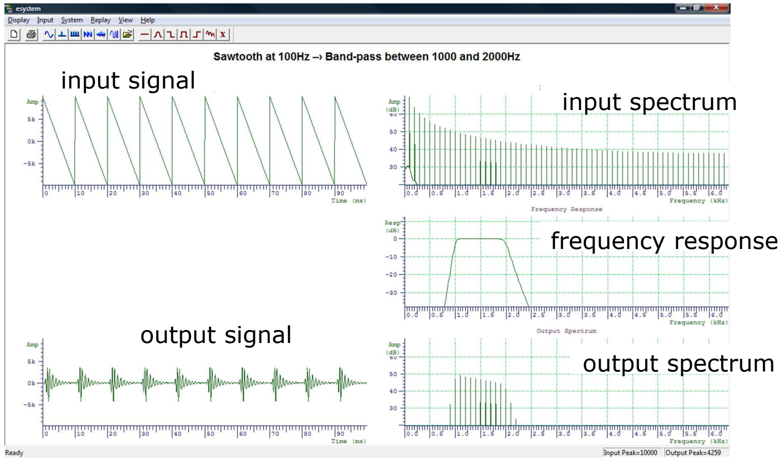
Using an aperiodic input (white noise): A *continuous* spectrum



Note that additions are done 'all at once'

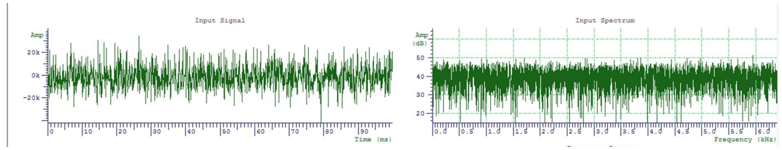
Consider this frequency response (what is it?)



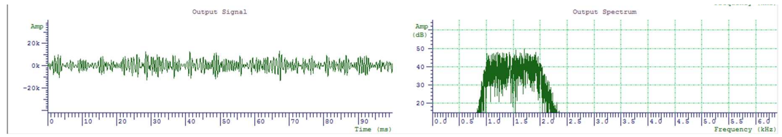


White Noise

IN

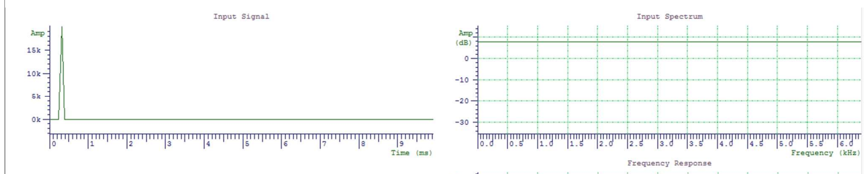


OUT

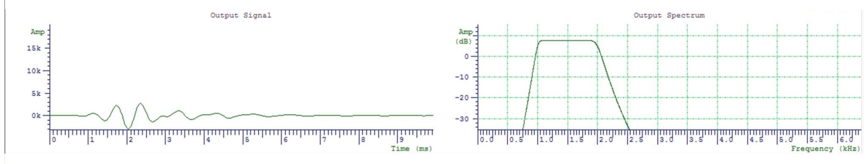


Single pulse

IN



OUT



More complex examples